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CALCULUS FOR SCHOOLS

By W. H. TYLER

The title of this paper may have various psychological reactions on the reader, according to his mental attitude and experience. To some it will perhaps seem as reasonable as teaching Greek in the kindergarten. A generation ago, when some of us were young, calculus was an ultimate goal of college mathematics, to be attained only after prolonged and painful servitude in the regions of college algebra and conic sections. The idea of teaching it even to college freshmen would have seemed heterodox, if indeed anybody had been so radical as to propose it. When finally reached it appealed only to the algebraically minded with its marvelous intricacies of partial fractions and reduction formulas. Excursions into geometry relieved the perplexities of some, to be sure, but the fact that calculus should be anything but an end in itself was more or less effectually disguised.

The completion of the subject was something to be celebrated with bon fires and illuminations, followed by very prompt oblivion of whatever had been inadvertently assimilated, except of course by the elect few who on the basis of congenital predilection for mathematics—and perhaps unfitness for anything else—were to continue into the fastnesses of differential equations and the like. Why then, in the name of all that is conservative and orthodox in education, talk about calculus for secondary schools?

A partial answer to this may be found in the history of elementary mathematics and its teaching. Our traditional school programs include—and have included for several generations—algebra through quadratic equations and the six books of plane geometry, but not many centuries ago the situation was notably different. In the sixteenth or seventeenth century, for example, education for boys of fourteen to eighteen would have included something resembling our grammar school arithmetic with protracted study of long division and a great deal of time on weights and measures, perhaps the algebra of the first degree equation and a little of the elements of plane geometry, the bulk of which was reserved for university students, together, of course, with trigonometry and the less elementary parts of algebra.

The evolution of the school program, in spite of inertia and powerful conservative tendencies, has consisted in the continual sifting of the inherited program, the progressive elimination of topics of minor importance and the bringing back of the essential parts of new material from the higher grades to the lower. It is unthinkable that this process should have reached the limit of its possibilities in our time. In our algebra, for example, there are many survivals of material which antedates the invention of the calculus and while not without considerable intrinsic interest for the algebraically minded, is of merely historical importance. The solution of the cubic equation, for example, was a great achievement of the sixteenth century and has a just claim on the interest of present day teachers of algebra, but that alone would be no sufficient reason for teaching it even in college algebra.

Again, in geometry, the Euclidean development while not inappropriate for an age when it constituted the whole body of mathematical training and when physics, chemistry and biology were unknown or unorganized, contains much that is non-essential in the modern development of mathematics.

Under favorable conditions at the present time there can be no fundamental difficulty in thoroughly teaching the essentials of arithmetic, algebra and geometry to boys and girls by the time they are sixteen. The question then is, as to the best use of available time for a year or two more, first as between mathematics and other subjects; second, as between various branches of mathematics. Is it best for a boy or girl to devote another year or two to mathematics, to spend it on additional algebra or on elementary analytic geometry, or on elementary calculus, or on some branch of applied mathematics, as simple surveying, etc.? No one answer to this question will be uniformly valid but in many cases the answer should unhesitatingly be the choice of the elementary calculus, particularly for pupils who are *not* continuing their education through colleges or scientific schools where the calculus would be inevitable.

The main reasons for this contention are:

First, the immense importance of the fundamental notions of limit and derivative without which a person is unable to form a clear and distinct conception of quantitative change in the world around him except in the crude sense of averages;

Second, the simplicity of the technique, if attention is confined to the polynomial functions with perhaps a few additions from the fractional or trigonometric;

Third, the importance of the notion of integration as a summation, and

Fourth, the comprehensiveness of calculus as built upon all the preceding mathematical subjects, and, on the other hand, as opening up unlimited possibilities of concrete application.

Granted all that has been said as to the value and importance of elementary calculus, is it practicable to teach it under secondary school conditions? This question must, of course, be finally answered on the basis of actual experience (to which the present writer can make no claim). Fortunately, however, the data on this side are fairly abundant. Elementary calculus has long been taught in the French lycée and the German gymnasium. It is now taught in English schools to boys of corresponding age. Of course it is necessary to adapt the teaching and the text to the age and capacity of the pupils.

These generalities may seem an inconclusive preamble without something more specific—which should, of course, presuppose experience. As to this, it is submitted that while there is doubtless a marked difference in capacity between the average high school senior and the average college freshman, there is no such gulf fixed between the best tenth—let us say—of the school seniors and the average freshman. It is for that tenth in the larger and stronger schools that the elements of the calculus may well be offered.

This does not mean a series of technical terms, elaborate definitions and rigorous proofs. Statements must be carefully framed and solecisms severely excluded, but the point of view may be largely intuitive and formal proofs subordinated or postponed. It is essentially a matter of studying the change in one quantity due to the change in another—and making that study more searching and exact than it can be made without the calculus.

A familiar—possibly too familiar—illustration is the automobile trap. The policeman knows no calculus, but he is well aware that the speed of a car at a station A in a distance S is not to be determined by merely dividing the total distance by the time T in which it was traversed. He must take a short interval

$AB (= \triangle S)$ and divide this by the corresponding short time $(\triangle t)$ in which *it* was covered. Even this will be only an average, less than the speed at A if, for example, the driver chanced to observe the policeman in time. The true speed at A can only be defined as the limit $\triangle s / \triangle t$ as $t \quad o$. The policeman is seeking a derivative—however innocently.

Again, in physics a boy learns that the motion of a stone thrown at an angle of elevation is expressed approximately by the equations $y = v_0 t \sin a - \frac{1}{2} g t^2$, $x = v_0 t \cos a$. How high will it rise, at what distance will it strike the ground and for what value of a will that distance be greatest. The possibility of solving the problem more laboriously without calculus detracts nothing from the value of the illustration.

The other great calculus concept of integration as a summation can be based on the study of simple problems in pressure upon vertical as contrasted with horizontal surfaces. If P is the total pressure above a certain horizontal line at a depth y, $\triangle P$ the pressure on the adjacent horizontal strip of width $\triangle y$ and area $\triangle A$ then $\triangle P$ lies between $Ky \triangle A$ and $K(y + \triangle y) \triangle A$, K denoting the density, so $P = K \int y \, dA$. The summation by integration will naturally have been approached by preliminary approximate summation without calculus, for example, finding the area of a river cross-section by the trapezoids which correspond to soundings at definite intervals. Much emphasis will be put on simple concrete problems in maxima and minima, selected with a view to avoiding algebraic complications. Contrast the value and interest of such work with the ordinary topics of advanced algebra, for which even the mathematician will rarely have future use. The study of the elementary calculus clarifies the idea of dependent variability—functionability—as nothing else can, and that idea pervades the quantitative activities of life.